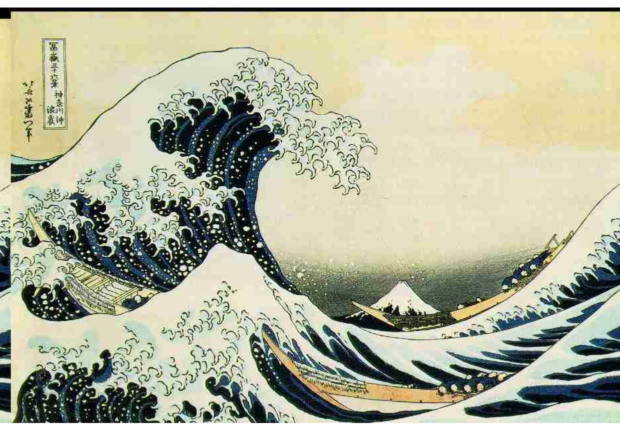
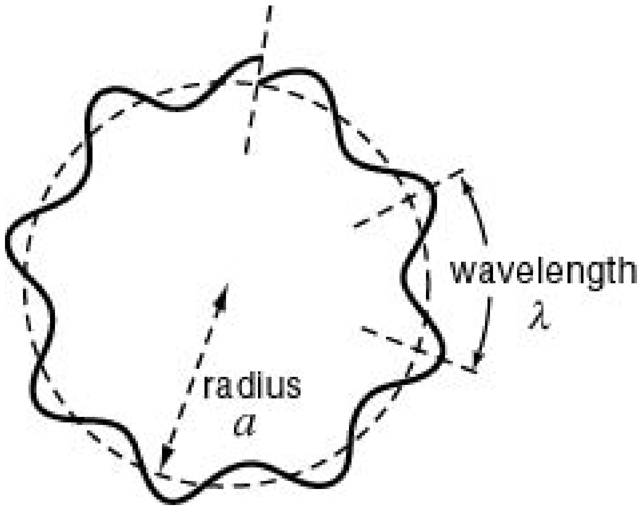
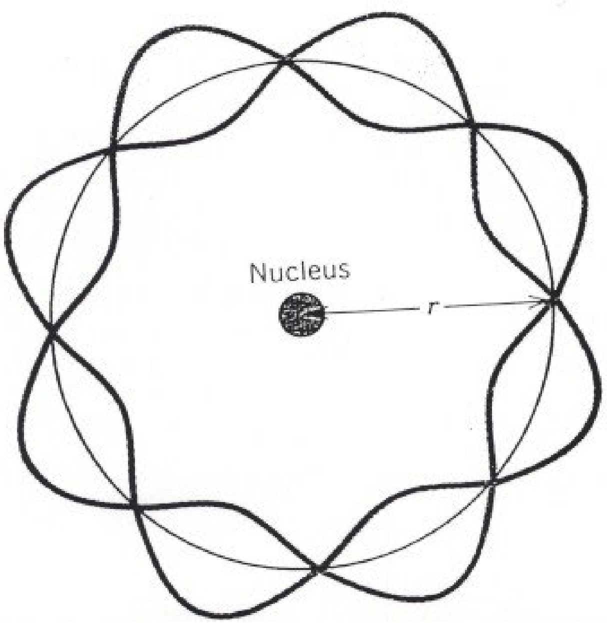
deBroglie “Matter Waves”

deBroglie and his matter waves, and its consequences
for physics and our concept of reality


Why quantization of angular momentum?





$$m_e vr = n\hbar \quad n = 1, 2, 3\ldots$$
$$\hbar = h / 2\pi$$

an integer number of wavelengths fits into the circular orbit

$$n\lambda = 2\pi r$$

where

$$\lambda = \frac{h}{p}$$

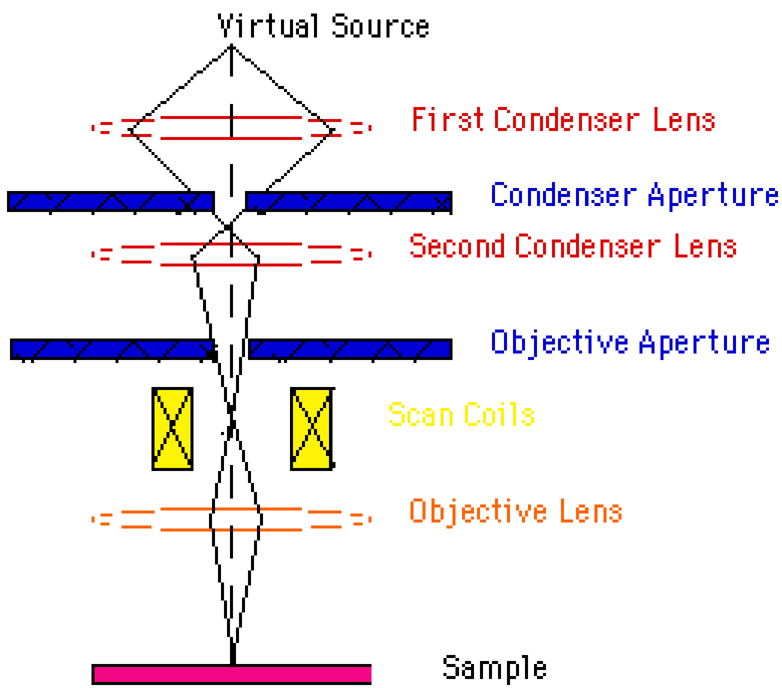
λ is the de Broglie wavelength

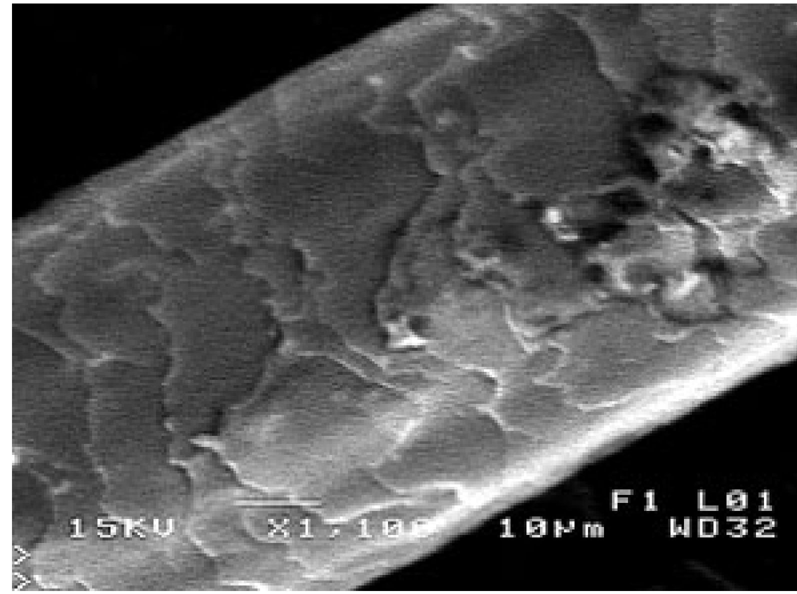
Magnitudes of deBroglie wavelengths

Particle		*Value of λ_{dB}*
Electrons of kinetic energy	1 eV	12.2 Å
	100 eV	1.2 Å
	10000 eV	0.12 Å
Protons of kinetic energy	1 keV	0.009 Å
	1 MeV	28.6 *fm*
	1 GeV	0.73 *fm*
Thermal neutrons (300K)		1.5 Å
Neutrons of kinetic energy 10 MeV		9.0 *fm*
He atoms at 300K		0.75 Å
you, walking to the student union for lunch at 2 miles per hour		2.54×10^{-34} m

$$1\ \text{Å} = 10^{-10} meters \qquad 1\ fm = 10^{-15} meters = 10^{-5}\text{Å}$$

Scanning Electron Microscope



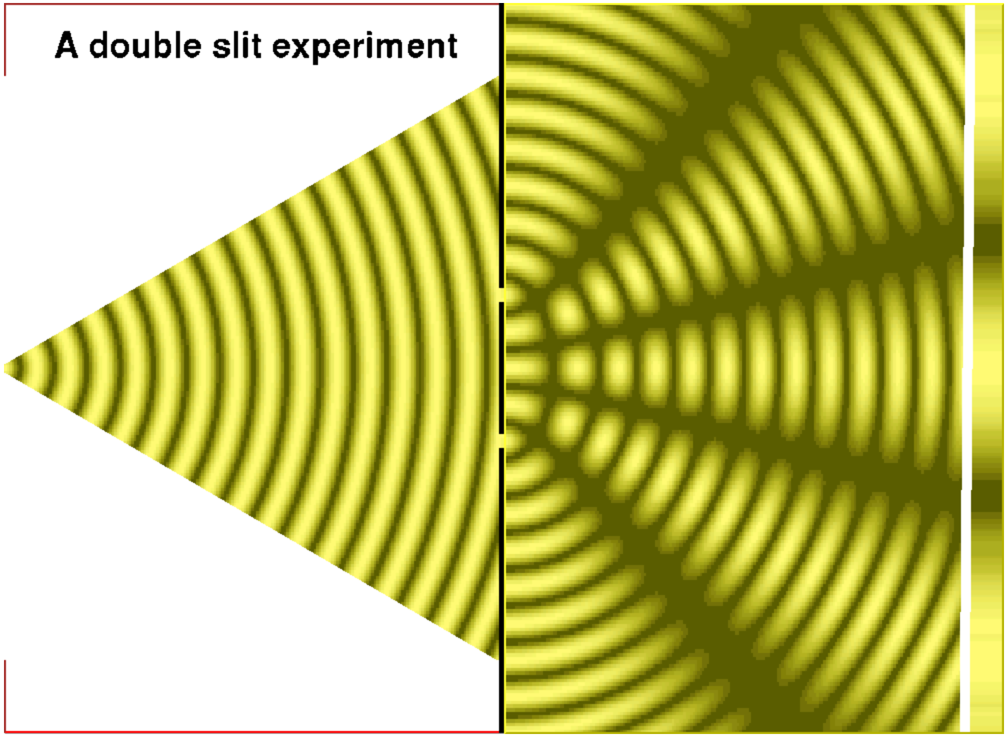


Human hair

Table Salt

Red Blood Cells

the realization that matter has wavelike properties

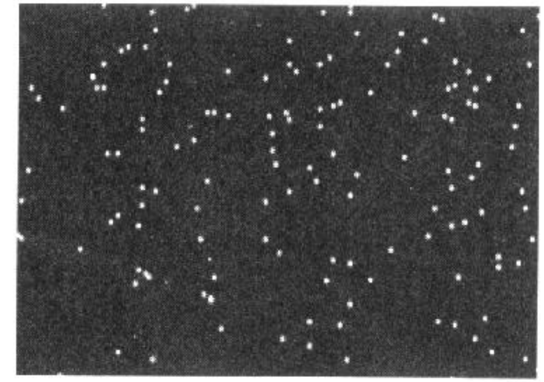


Wave Properties of Electrons

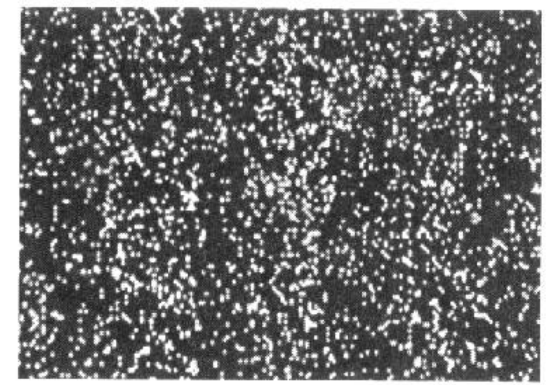
Transmission Electron Microscope

Electrons passed through two slits and detected

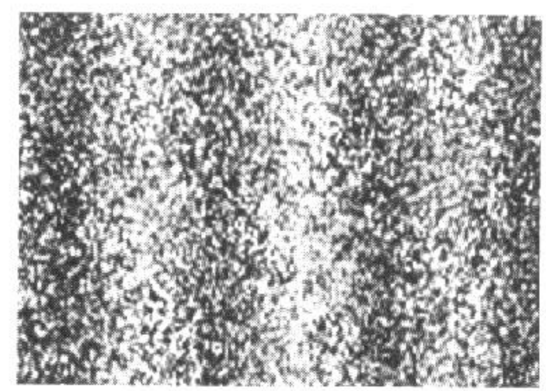
http://www.hitachi.com/rd/portal/highlight/quantum/doubleslit/index.html



(*b*) After 100 electrons



(*c*) After 3000 electrons



(*d*) After 70 000 electrons

What does it mean?

•A large number of electrons going through a double slit will produce an interference pattern, like a wave.

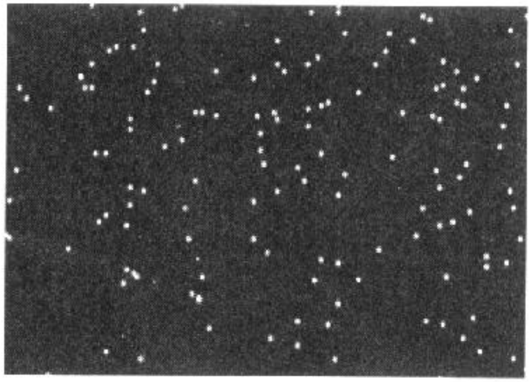
•However, each electron makes a single impact on a phosphorescent screen-like a particle.

•Electrons have indivisible (as far as we know) mass and electric charge, so if you suddenly closed one of the slits, you couldn’t chop the electron in half-because it clearly is a particle.

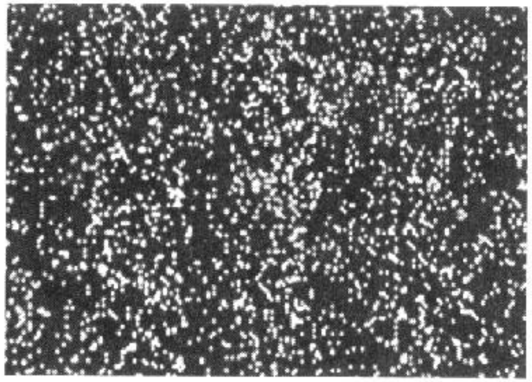
•A large number of electrons fired at two simultaneously open slits, however, will eventually, once you have enough statistics, form an intereference pattern. Their cumulative impact is wavelike.

•This leads us to believe that the behavior of electrons is governed by probabilistic laws. --The wavefunction describes the probability that an electron will be found in a particular location.

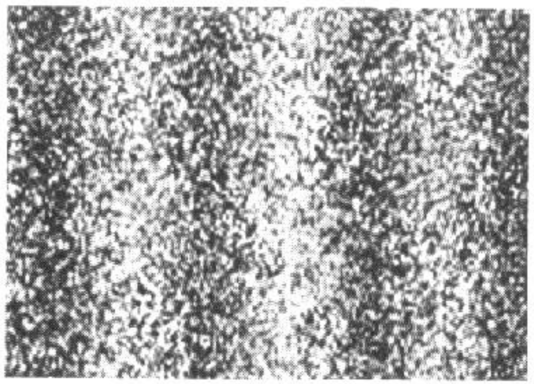
 (see animation https://www.youtube.com/watch?v=Xmq_FJd1oUQ)



(b) After 100 electrons



(c) After 3000 electrons



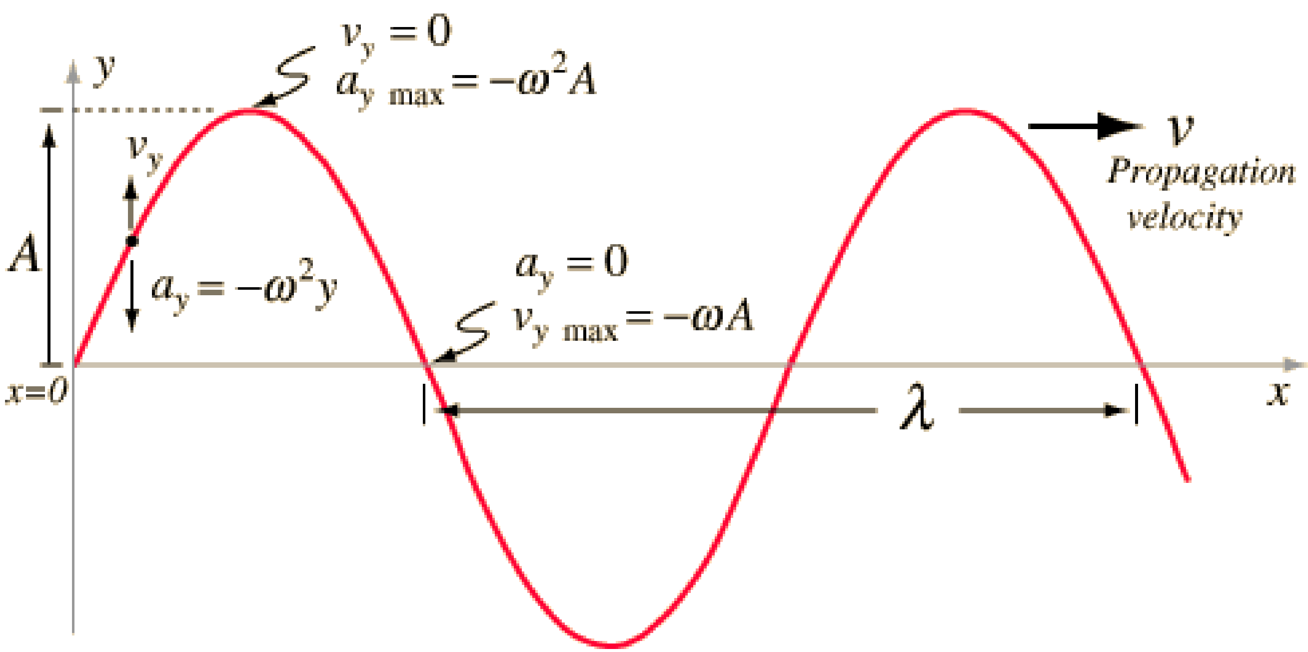
(d) After 70 000 electrons

We are about to embark on a wave theory of matter …

It is a good idea to review some concepts about waves

Anatomy of a classical travelling wave

For example:
Wave on a string
Snapshot at some time t



Description of the transverse motion.

$$\frac{2\pi v}{\lambda} = 2\pi f = \omega$$

$$v = f\lambda$$

$$y(x,t) = A\sin\frac{2\pi}{\lambda}(x - vt)$$

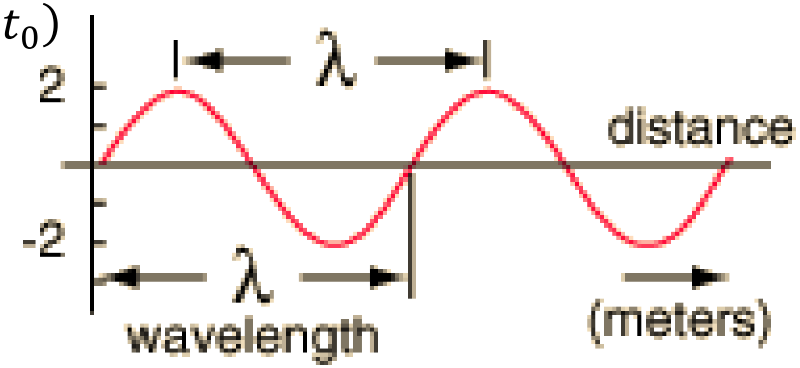
$$v_y(x,t) = \frac{dy}{dt} = \omega A\cos\frac{2\pi}{\lambda}(x - vt)$$

$$a_y(x,t) = \frac{d^2y}{dt^2} = -\omega^2 y = -\omega^2 A\sin\frac{2\pi}{\lambda}(x - vt)$$

Anatomy of a classical travelling wave

Fixed time (snapshot)

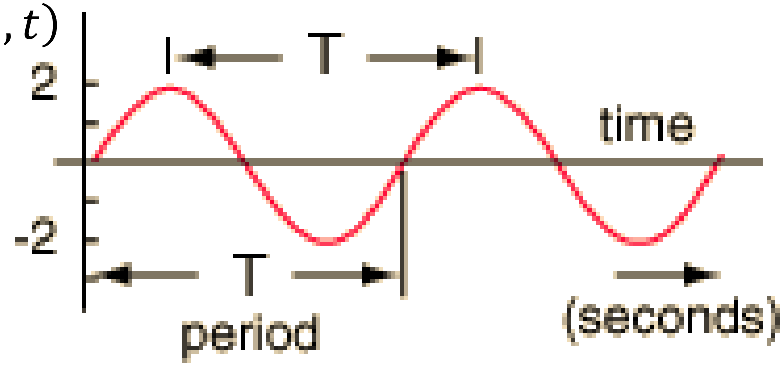
$y(x, t = t_0)$



x

Fixed location

$y(x = x_0, t)$



t

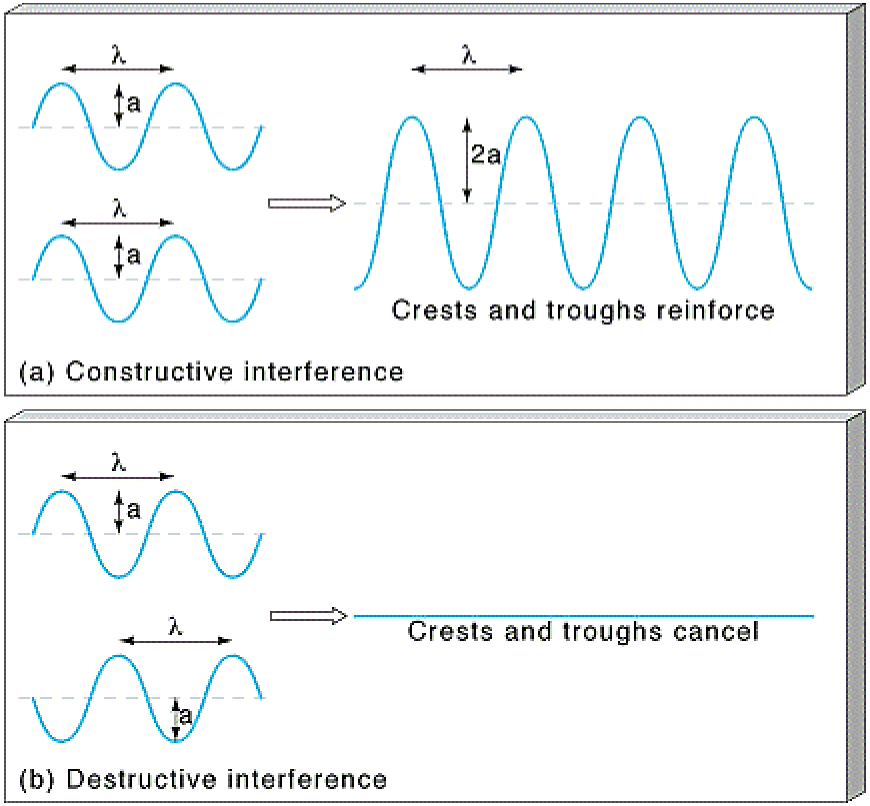
$$y(x,t) = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

Wavenumber: $k \equiv \dfrac{2\pi}{\lambda}$

All harmonic waves satisfy: $f\lambda = v$ $\qquad f = v/\lambda$

so $2\pi f = \omega = \dfrac{2\pi v}{\lambda} = kv$

Hence $\boldsymbol{y(x,t) = A\sin(kx - \omega t)}$



Interference

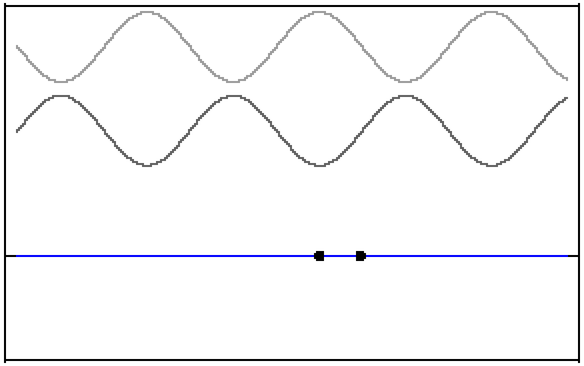
waves can interfere (add or cancel)

Two solutions to a **linear** wave equation:

$y_1(x,t), y_2(x,t)$

Any linear combination is also a solution to the wave equation:

$y_{super}(x,t) = ay_1(x,t) + by_2(x,t)$

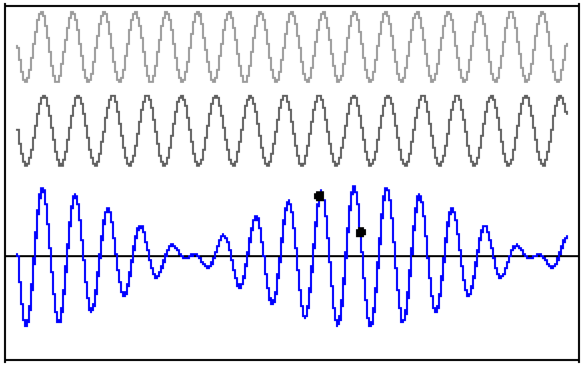


Interefering waves, generally…

$$y = y_1 + y_2 = A\cos(k_1 x - \omega_1 t) + A\cos(k_2 x - \omega_2 t)$$

$$\Downarrow$$

$$y = 2A\cos\frac{1}{2}\{(k_2 - k_1)x - (\omega_2 - \omega_1)t\} \bullet \cos\frac{1}{2}\{(k_1 + k_2)x - (\omega_1 + \omega_2)t\}$$



"Beats" occur when you add two waves of slightly different frequency. They will interfere constructively in some areas and destructively in others.

Can be interpreted as a sinusoidal envelope: $2A\cos\left(\frac{\Delta k}{2}x - \frac{\Delta\omega}{2}t\right)$

Modulating a high frequency wave within the envelope: $\cos\left[\frac{1}{2}(k_1 + k_2)x - \frac{1}{2}(\omega_1 + \omega_2)t\right]$